

College Board Standards for College Success™

Mathematics and Statistics

*Three-Year Alternative Standards for
Middle School Mathematics*

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Three-Year Alternative Standards for Middle School Mathematics

In the Mathematics and Statistics College Board Standards for College Success™, the content outlined for Middle School Mathematics I and II is presented as a two-year course of study. The following set of standards is offered as an alternative for school systems whose middle school programs allow students to complete this material over a three-year period. The three courses in this alternative set of middle school standards will be referred to as Middle School Mathematics A, B, and C to alleviate confusion with the Middle School Mathematics I and II as outlined for the two-year curriculum.

While the overall goals of the middle school curriculum outlined in Mathematics I and II are the same, the more relaxed pace of the courses and the increased class time available give students greater opportunities to acquire the knowledge and skills associated with the standards in Mathematics A, B, and C. The tradeoff for this added time and opportunity to achieve mastery of the content is that students will not have the opportunity to take Advanced Placement (AP®) Calculus in their senior year of high school, unless they double up on coursework during one year of high school or complete one of the high school courses in a summer-session program.

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Middle School Math A

STANDARDS

1. Nonnegative Rational Numbers [Number and Operations]¹
2. Ratios and Rates [Number and Operations]
3. Two-Dimensional Geometry and Measurement [Geometry and Measurement]
4. Univariate Categorical Data Analysis [Data Analysis]

Course Goal:

Students compute fluently with nonnegative rational numbers and have facility with their application in mathematical and real-world settings. Students reason with rates and ratios to solve problems. Students classify and sketch planar figures, as well as derive their perimeters and areas. Students develop displays for sets of categorical data, describing the center and spread of such sets.

Math A corresponds closely to grade 6 in a traditional grade-level program. The course focus is on completing the majority of work associated with nonnegative decimals and fractions. Students encounter ratios and rates and use them to represent and reason about comparisons, including percents. They consider the notions of rates and ratios, using them in structuring and solving proportion problems, especially those involving direct comparisons of amounts.

Students apply the concepts of parallelism, perpendicularity, and symmetry in classifying and sketching familiar two-dimensional figures. Math A students measure or use measurements to find the perimeter and area of rectangles and triangles.

Math A students investigate simple questions that can be addressed by collecting univariate categorical data about a small population. They develop displays showing categorical data in graphs or tables, and they interpret the relationships shown in such displays.

The following mathematics standards, objectives, and performance expectations do not represent any specific curricular design. They are presented to provide guidance to curriculum and instructional designers as they connect and integrate this content into effective instruction.

¹ The category mentioned in the brackets refers to corresponding content in the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (2000).

Standard MA.1: Nonnegative Rational Numbers

Students develop number sense* related to nonnegative rational numbers and whole-number exponents. Students apply the concepts, properties, and operations associated with nonnegative rational numbers in solving multistep word problems. Students establish fluency in developing equivalent representations of and computing with these numbers as appropriate to given problem situations.

Objective MA.1.1:

Student develops number sense encompassing magnitude, comparison, order, and equivalent representations, which supports reasoning in operating with nonnegative rational numbers in fraction and decimal forms. Student applies these concepts, operations, and properties in solving problems involving relationships among whole numbers and other nonnegative rational numbers.

Performance Expectations:

- MA.1.1.1 Represents, models, compares, and orders nonnegative rational numbers using graphical, pictorial, concrete, and numerical representations, including the use of equivalent fractions.
- MA.1.1.2 Identifies and creates problem situations involving addition, subtraction, multiplication, and division of nonnegative rational numbers.
- MA.1.1.3 Applies the properties associated with nonnegative rational numbers, including both their fraction and decimal representations, in solving problems.

Objective MA.1.2:

Student becomes fluent in finding equivalent representations of, estimating, and computing with nonnegative rational numbers, in fraction and decimal forms, as appropriate to given problem situations.

Performance Expectations:

- MA.1.2.1 Identifies and develops equivalent representations of nonnegative rational numbers, and translates fluently among these representations to fit a context or multistep problem situation, recognizing whether the numbers involved are reasonable.

*Underlined words and phrases are defined in the Glossary that appears in the full Mathematics and Statistics College Board Standards for College Success.

- MA.1.2.2 Selects and applies an appropriate fraction or decimal representation for nonnegative rational numbers in a given context.
- MA.1.2.3 Selects an appropriate approach (estimate and check, mental math, paper-and-pencil, or technology) and applies it to solve a computational problem involving nonnegative rational numbers.
- MA.1.2.4 Computes fluently in situations involving addition, subtraction, multiplication, and division of nonnegative rational numbers in fraction and decimal forms.
- MA.1.2.5 Estimates results involving nonnegative rational numbers, and judges the reasonableness of both one's own and others' estimates.

Objective MA.1.3:

Student develops a number sense related to natural-number exponents, including knowledge of powers, multiples, divisors, factors, primes, and composites. Student understands and applies the prime factorization of positive integers in solving problems.

Performance Expectations:

- MA.1.3.1 Reads, writes, and evaluates expressions involving natural-number powers of positive integers.
- MA.1.3.2 Makes observations, conjectures generalizations, and provides plausible explanations about numerical relationships, such as the divisibility rules for 2, 3, 5, 9, and 10.
- MA.1.3.3 Explains and applies the Fundamental Theorem of Arithmetic to represent numbers as products of prime factors.
- MA.1.3.4 Uses the prime factorization of two or three natural numbers to find their greatest common divisor/factor and least common multiple.

Standard MA.2: Ratios and Rates

Students develop the ability to use ratios and rates to represent and reason about comparisons and to solve multistep problems, including those involving percent and those involving conversion of measures within the same measurement system.

Objective MA.2.1:

Student identifies and represents ratios and rates as comparisons, and reasons to find equivalent ratios to solve problems involving proportional relationships.

Performance Expectations:

- MA.2.1.1 Identifies one or more ratios that represent a given comparison, and expresses the ratios using appropriate notation (i.e., $\frac{a}{b}$, a/b , a to b , $a : b$).
- MA.2.1.2 Describes the use of equivalent ratios in proportional relationships, and generates and applies appropriate equivalent ratios in problem situations involving scales and measurement conversion factors, percents, and probabilities.

Objective MA.2.2:

Student develops computational fluency in working with ratios, percents, and proportional situations, and applies this fluency to estimate the solution to and solve a variety of real-world problems.

Performance Expectations:

- MA.2.2.1 Uses unit rates and equivalent ratios to represent and make quantitative comparisons in solving problems in real-world contexts (e.g., equivalent measures, discounts, interest, taxes, tips).
- MA.2.2.2 Represents and models ratios associated with whole-number percents that are less than or equal to 100%.
- MA.2.2.3 Mentally estimates and finds solutions to percent problems.
- MA.2.2.4 Translates fluently among the various representations for fractions, decimals, and percents in appropriate ways.

Standard MA.3: Two-Dimensional Geometry and Measurement

Students identify, classify, and represent figures using the relations of parallelism, perpendicularity, and symmetry. Students develop and apply formulas for measurements in the plane, especially those involving the perimeter/circumference and area of triangles, quadrilaterals, and circles, and composite figures made from these shapes.

Objective MA.3.1:

Student represents geometric figures from written or verbal descriptions, measurements, and properties using sketches, figures on grids, or models and applies parallelism, perpendicularity, and symmetry in real-world settings.

Performance Expectations:

- MA.3.1.1 Uses definitions and properties of two-dimensional figures to classify figures, and sketches figures having given properties.
- MA.3.1.2 Sketches two-dimensional figures with specified measures, both in rough sketch form and on graph paper.
- MA.3.1.3 Describes and applies the relationships of parallelism, perpendicularity, and symmetry in real-world settings involving two dimensions.

Objective MA.3.2:

Student distinguishes between length and area contexts, develops an understanding of formulas, and applies them to find the perimeter/circumference and area of triangles, quadrilaterals, circles, and composite figures made from these shapes.

Performance Expectations:

- MA.3.2.1 Distinguishes between appropriate units for linear and area measurement situations.
- MA.3.2.2 Describes the relationship between the circumference and diameter of a circle, $\pi = \frac{C}{d}$, and applies it to develop convincing arguments about the validity of formulas such as $C = 2\pi r$ and $A = \pi r^2$.
- MA.3.2.3 Finds the area of triangles, parallelograms, and trapezoids, by measuring or using measurement of sides and altitudes, by counting and rearranging portions of squares in a grid, and by developing and using formulas.
- MA.3.2.4 Uses given information to find the perimeter and area of a composite figure by dividing it into known shapes and using the measures of these shapes to determine the measures of the composite figure.

Standard MA.4: Univariate Categorical Data Analysis

Students formulate and investigate simple questions that can be addressed by collecting univariate categorical data about a small population. They collect, organize, display, and interpret such univariate data to answer the questions posed.

Objective MA.4.1:

Student formulates a question about one small population that can be answered through collection of univariate categorical data, describes the data collection process, and collects data. Student organizes and displays the data and interprets the data displays.

Performance Expectations:

- MA.4.1.1 Formulates a simple question, and defines a small population on which univariate categorical data can be collected to answer the question.
- MA.4.1.2 Determines a data collection process and collects the data.
- MA.4.1.3 Constructs frequency distributions and other appropriate graphical displays (bar graphs), and determines the mode for categorical data.
- MA.4.1.4 Interprets results and communicates conclusions in the context of the formulated question using appropriate symbols, notation, and terminology.

Middle School Math B

STANDARDS

1. Integers to the Rational Numbers [Number and Operations]
2. Two- and Three-Dimensional Geometry [Geometry]
3. Similarity and Measurement [Geometry and Measurement]
4. Linear Patterns and Relationships [Algebra]
5. Univariate Numerical Data Analysis [Data Analysis]
6. Experimental and Theoretical Probability [Probability]

Course Goal:

Students reason and compute fluently with integers and rational numbers and apply them in solving problems, including those related to one-stage probability experiments. Students reason with proportional relationships to structure and solve problems in two-dimensional geometry and similarity, and they represent three-dimensional figures in terms of related two-dimensional figures. Students develop the ability to formulate a question that can be addressed by collecting univariate numerical data about a small population. They collect, represent, and interpret data in answering that question. Students examine a one-stage probability problem from theoretical and experimental standpoints.

Students in Math B continue the transition from arithmetic to algebra through the study of the integers and their operations. This knowledge allows them to extend their understanding of rational numbers to include both negative and nonnegative rational numbers and operations on them. Students in Math B also encounter integer exponents.

Math B students' evolving knowledge of the integers allows them to represent and describe the locations of points in all four quadrants of the plane. These students investigate the effects of rigid transformations on figures in the coordinate plane, and they explore the symmetries of planar figures. Students investigate and connect topics in number, geometry, measurement, probability, and algebra through the lens of proportionality. A significant amount of time is spent on the development of similar figures and reasoning about geometric figures to develop properties and measurement formulas.

In their study of content from algebra, Math B students examine and graph patterns showing linear relations, develop expressions with nonnegative rational numbers as coefficients to symbolize linear patterns, and represent these patterns and relationships with graphs, tables, and verbal rules. Students learn how to evaluate linear expressions and solve one-step linear equations.

Math B students work on developing the ability to formulate a simple question about one small population, or about a comparison between two small populations, that can be answered by collecting and analyzing univariate numerical data. They design straightforward data investigations, collect and analyze data, and use their analyses to answer the question they have formulated.

Math B students investigate simple one-stage experiments empirically and, where possible, compare theoretical and experimental probabilities. They develop a concept of probability as a ratio describing successes or failures to total trials and having a value between 0 and 1, inclusive.

In Math B, students have occasions to understand the mathematical concepts they study using different representations in various contexts. The connections between these representations are studied explicitly and are also encountered in problem-solving situations. Understanding such translations is central to the development of students' abilities in solving problems and communicating results.

The following mathematics standards, objectives, and performance expectations do not represent any specific curricular design. They are presented to provide guidance to curriculum and instructional designers as they connect and integrate this content into effective instruction.

Standard MB.1: Integers to the Rational Numbers

Students extend their number and operation sense to the entire set of integers and to the negative rational numbers, their varied representations, their operations, and their properties. Students develop fluency in computing with integers and rational numbers, including the decimal forms of rational numbers, and develop an intuitive understanding of the addition of integers and additive inverses. Students reason about and solve problems involving integers and rational numbers.

Objective MB.1.1:

Student models operations, computes fluently, and solves problems with integers.

Performance Expectations:

- MB.1.1.1 Models the addition, subtraction, multiplication, and division of integers, describes the relationships among these operations, and applies the order of operations.
- MB.1.1.2 Computes fluently with integers, including identifying roots of perfect squares and perfect cubes.
- MB.1.1.3 Represents and solves mathematical and real-world problems involving integers.
- MB.1.1.4 Estimates and judges the reasonableness of results involving integer operations.

Objective MB.1.2:

Student develops an understanding of the concepts of order, additive inverses, and addition of integers and solves simple addition problems involving integers.

Performance Expectations:

- MB.1.2.1 Describes the extension to the integers of concepts of order and position, and confirms the properties of addition of whole numbers (e.g., closure, associative, commutative, identity) for the addition of integers.
- MB.1.2.2 Recognizes and formulates problems involving the addition of integers, and solves them using number lines, patterns, and models.
- MB.1.2.3 Locates and plots points whose coordinates are both integers on the four-quadrant coordinate plane.

MB.1.2.4 Recognizes and applies the absolute value of an integer as its distance from zero on a number line.

Objective MB.1.3:

Student computes fluently with rational numbers written in fraction and decimal forms, and student solves problems involving rational numbers.

Performance Expectations:

MB.1.3.1 Describes and applies the relationships characterized by

$$a - b = a + (-b) \text{ and } a \div b = a \cdot \frac{1}{b}.$$

MB.1.3.2 Computes fluently with all rational numbers in both fraction and decimal form.

MB.1.3.3 Represents and solves problems involving rational numbers, and judges the reasonableness of the solutions.

Standard MB.2: Two- and Three-Dimensional Geometry

Student formulates general statements relating two- and three-dimensional figures by relevant characteristics and properties, and student makes conjectures concerning general properties and transformations of specified figures.

Objective MB.2.1:

Students formulate general statements defining and relating two- and three-dimensional figures using their relevant characteristics and geometric properties. Students recognize and apply properties of angles and figures, as well as the relationship between rigid transformations of figures and congruence of figures, in representing these figures with and without coordinates.

Performance Expectations:

MB.2.1.1 Formulates general statements describing properties of circles, polygons, prisms, pyramids, cones, spheres, and right-circular cylinders.

MB.2.1.2 Relates and applies planar nets in analyzing and representing three-dimensional figures in terms of related two-dimensional figures.

- MB.2.1.3 Represents three-dimensional figures by sketches and by using isometric dot paper*
- MB.2.1.4 Identifies and graphs points in all four quadrants of the coordinate plane, and draws and labels the vertices of basic shapes on the plane.
- MB.2.1.5 Locates and gives the resulting transformed coordinates of a figure produced after a translation, a reflection about a vertical or horizontal line, or a rotation of a multiple of 90° about the origin of the coordinate plane.
- MB.2.1.6 Makes and supports conjectures about general properties of figures, such as the 180° angle sum for triangles, measures of the opposite angles in parallelograms, measures of vertical angles, and angle relationships in common shapes and figures.

Standard MB.3: Similarity and Measurement

Students develop and apply similarity relationships to solve problems. Students apply the concepts of scaling methods to solve problems involving drawings and maps.

Objective MB.3.1:

Student identifies, describes, and applies similarity relationships to find measures of corresponding parts in similar figures and applies scales to measurements in drawings and maps.

Performance Expectations:

- MB.3.1.1 Defines and identifies similarity of two-dimensional figures, including the linking of corresponding parts, the similarity ratio, and the measures of corresponding parts.
- MB.3.1.2 Determines a proportional relationship among measures of corresponding sides of similar figures.
- MB.3.1.3 Solves indirect measurement problems and scaling problems involving mathematical and real-world contexts using similar figures, with or without grids.
- MB.3.1.4 Interprets and solves situations using scales, including those based on number lines, drawings, models, maps, and graphs.

*Underlined words and phrases are defined in the Glossary that appears in the full Mathematics and Statistics College Board Standards for College Success.

Standard MB.4: Linear Patterns and Relationships

Students identify linear patterns for mathematical and real-world situations and represent linear expressions in words, tables, symbols, and graphs. Students write and solve one-step linear equations.

Objective MB.4.1:

Student creates and evaluates simple linear expressions to represent linear patterns and develops graphs to represent these expressions.

Performance Expectations:

- MB.4.1.1 Represents linear patterns generated by mathematical and real-world situations with expressions, and evaluates these expressions for nonnegative rational numbers.
- MB.4.1.2 Generates and graphs a set of ordered pairs representing a given linear expression.
- MB.4.1.3 Writes rules in words and in symbols for situations modeled by the forms ax and $x \pm b$, where a and b are nonnegative rational numbers.
- MB.4.1.4 Extends linear patterns drawn from mathematical and real-world settings, sequences, tables, and graphs with verbal or symbolic rules of the form $ax \pm b$, where a and b are nonnegative rational numbers.

Objective MB.4.2:

Student uses linear patterns to create simple linear equations, and student solves these equations.

Performance Expectations:

- MB.4.2.1 Uses linear patterns drawn from real-world settings to create equations and solve problems.
- MB.4.2.2 Evaluates nonnegative rational numbers as possible solutions to linear equations numerically and graphically, with and without technology.
- MB.4.2.3 Writes and solves linear equations of the form $ax = b$ and $x \pm b = c$, where a , b , and c and the solution are nonnegative rational numbers.

Standard MB.5: Univariate Numerical Data Analysis

Students formulate and answer questions about small populations by collecting and analyzing univariate numerical data from the populations. Students draw conclusions from their analyses and communicate their results, and they identify flaws in faulty presentations of data found in the media.

Objective MB.5.1:

Student formulates a question about one small population or about a comparison between two small populations that can be answered through collection and analysis of univariate numerical data, designs related data investigations, and collects data.

Performance Expectations:

- MB.5.1.1 Formulates a simple question, and defines one or two small populations on which data can be collected to answer the question.
- MB.5.1.2 Identifies a numerical attribute on which to collect data, decides how to measure the attribute to answer the question formulated, determines a data collection process, and collects the data.
- MB.5.1.3 Recognizes and describes the differences between numerical data and categorical data.
- MB.5.1.4 Constructs appropriate graphical displays (bar graphs, line plots, stem-and-leaf plots, histograms), with and without technology, to describe the distribution of data values.
- MB.5.1.5 Describes the shape, center, and spread of the distribution of numerical data.
- MB.5.1.6 Computes measures of center (mean, median) and spread (range) for a set of numerical data, with and without technology, interprets the meaning of these measures in context, and explains the influences of outliers on each measure.

Objective MB.5.2:

Student interprets results and communicates conclusions regarding a formulated question using appropriate symbols, notation, and terminology. Student identifies flaws in faulty or misleading presentations of data found in the media.

Performance Expectations:

- MB.5.2.1 Interprets results and communicates conclusions in the context of the formulated question using appropriate symbols, notation, and terminology.

MB.5.2.2 Identifies flaws in faulty or misleading presentations of data found in the media and their potential effects on conclusions drawn.

Standard MB.6: Experimental and Theoretical Probability

Students estimate probabilities in experiments and, where possible, compare experimental and theoretical probabilities.

Objective MB.6.1:

Student estimates probabilities in experiments with common objects, including comparing experimental and theoretical probabilities and examining experimental probabilities in the long run.

Performance Expectations:

- MB.6.1.1 Conducts experiments to estimate the likelihood of a simple event; compares the experimental probability with an easily identifiable theoretical probability, where the sample space is described by tables or lists of possible outcomes.
- MB.6.1.2 Uses the probabilities of events to compare the likelihoods of events. Describes why the probability of an event is a number between 0 and 1, inclusive.
- MB.6.1.3 Uses experimental data with graphical or tabular displays to estimate the probability of an event for which the theoretical probability is unknown.
- MB.6.1.4 Recognizes that repetitions of an experiment may result in different outcomes, describes the variation in outcomes produced in an experiment, and recognizes that with the collection of more data the experimental probability of a particular outcome approaches the theoretical probability.

Middle School Math C

STANDARDS

1. Rational Numbers to Real Numbers [Numbers and Operations]
2. Geometric Scale and Transformations [Geometry]
3. Pythagorean Theorem and Measurement of Three-Dimensional Objects [Geometry and Measurement]
4. Linear Equations and Inequalities [Algebra]
5. Bivariate Data [Data Analysis]
6. Probabilities in One-Stage Experiments [Probability]

Course Goal:

Students reason with properties of real-number operations and develop an intuitive understanding of irrational numbers such as π and $\sqrt{2}$. Students employ proportional relationships to structure and solve measurement problems in two- and three-dimensional geometry and to connect rate of change, slope, direct variation, linear relationships, and similarity on the coordinate plane. Students compute fluently with integers and rational numbers and apply them in solving problems, including those related to one-stage probability experiments. Students represent and interpret sets of bivariate data and use the associated skills to investigate and draw conclusions about related questions.

The focus on number changes from numbers and operations to systems and structures. Students in Math C view real numbers as the set of all decimal numbers and investigate the relationship between the format of decimals and the type of numbers they represent. Students extend their work with exponents to include negative-integer exponents and their use in representing numbers in scientific notation. Math C students also work to develop fluency in estimating with computations involving integers, rational numbers, and decimals.

The study of geometry and measurement concepts in Math C involves the extension of the concepts of perimeter and area in the plane to the study of surface area and volume and the related formulas for three-dimensional figures. Significant attention is given to the development of students' spatial visualization skills.

A significant amount of time is spent considering the notions of rates and ratios, and structuring and solving proportion problems, especially those involving direct variation. These topics are mirrored in geometry in the development of similar figures and the Pythagorean theorem, and the examination of and reasoning about geometric figures to

develop properties and measurement formulas. These investigations allow a discussion of irrational numbers, such as π and $\sqrt{2}$, and the system of real numbers.

The data analysis processes in Math C focus on situations where two pieces of data are collected from each member of a small population. Students develop their abilities to display and summarize bivariate data and to interpret and communicate their results. Students investigate simple one-stage experiments empirically and, where possible, compare theoretical and experimental probabilities.

Students learn to write algebraic expressions describing growth to deal with patterns that are linear (arithmetic) and to investigate the nature of growth in exponential (geometric) patterns. Specific emphasis is placed on developing students' abilities to represent linear relationships in a wide variety of formats, as well as interpreting the meaning of rate of change in different contexts. Within algebra, as well as across the other areas of Math C, students have many opportunities to interpret contextualized situations using a number of different representations. These opportunities are studied explicitly, as well as being included within problem-solving situations. The capability to translate among representations encountered in pattern or data situations is central to the development of students' abilities to solve problems and to communicate results.

The following mathematics standards, objectives, and performance expectations do not represent any specific curricular design. They are presented to provide guidance to curriculum and instructional designers as they connect and integrate this content into effective instruction.

Standard MC.1: Rational Numbers to Real Numbers

Students view real numbers as decimals and use positive- and negative-integer exponents and scientific notation to represent numbers. Students develop fluency in estimating real-number operations and use properties of operations in solving problems. Students reason with ratios, rates^{*}, percents, and proportional relationships to solve problems and interpret results.

Objective MC.1.1:

Student describes the real numbers as the set of all decimal numbers and uses scientific notation, estimation, and properties of operations to represent and solve problems involving real numbers. Students develop fluency in estimating and reasoning about real-number operations using properties of operations to represent and solve problems.

Performance Expectations:

- MC.1.1.1 Describes the real numbers as the set of all possible decimal numbers, and recognizes that representations of π , $\sqrt{2}$, and other irrational numbers are nonterminating, nonrepeating decimals.
- MC.1.1.2 Estimates results for mathematical and real-world problems involving operations and approximations with rational numbers, decimals, and percents, and judges the reasonableness of such computations and approximations.
- MC.1.1.3 Applies properties of real-number operations (associative, commutative, identity, inverse, distributive, closure, properties of equality and inequality) to solve a problem situation involving real numbers.
- MC.1.1.4 Reads, writes, and orders numbers represented in scientific notation using positive- and negative-integer exponents for powers of 10, and interprets applications of scientific notation in varied contexts including technological output formats.

Objective MC.1.2:

Student reasons with ratios, rates, percents, and proportional relationships to solve problems and interpret results.

^{*}Underlined words and phrases are defined in the Glossary that appears in the full Mathematics and Statistics College Board Standards for College Success.

Performance Expectations:

- MC.1.2.1 Interprets the meanings of rates of change associated with increases and decreases within mathematical and real-world contexts involving rates, ratios, measurement conversions within systems, scale drawings, and percents, and solves related problems.
- MC.1.2.2 Interprets and solves conversion problems involving scale drawings and constants of proportionality, and calculates derived measures such as density, foreign monetary equivalences, and those arising in other contexts.
- MC.1.2.3 Reasons about, structures, and solves problems involving rates, ratios, proportions, or percents, including percents less than 1 and greater than 100.

Standard MC.2: Geometric Scale and Transformations

Students identify, justify, and apply angle relationships in describing properties of geometric figures and relationships. Students relate and apply knowledge of rigid transformations.

Objective MC.2.1:

Student identifies, justifies, and applies angle measurements and relationships in describing geometric figures and relationships among their components.

Performance Expectations:

- MC.2.1.1 Reasons using models and drawings to construct and support convincing arguments about angle relationships in figures.
- MC.2.1.2 Identifies, states, and applies the basic properties associated with complementary angles and angles formed by transversals intersecting pairs of parallel lines.
- MC.2.1.3 Identifies, states, and applies the angle-sum properties for triangles and other polygons.

Objective MC.2.2:

Student relates and applies knowledge of rigid transformations in relating objects and in their effects on figures.

Performance Expectations:

- MC.2.2.1 Describes the effects of rigid transformations (translations, reflections about a vertical or horizontal line, rotations about the origin, and simple compositions of these transformations) on figures in the coordinate plane.
- MC.2.2.2 Uses rigid transformations to identify the corresponding parts of congruent figures.

Standard MC.3: Pythagorean Theorem and Measurement of Three-Dimensional Objects

Students develop and apply the Pythagorean theorem to solve measurement problems. Students apply the concepts of surface area and volume to measure three-dimensional figures.

Objective MC.3.1:

Student develops and applies the Pythagorean theorem to solve for the lengths of sides in right triangles and related measurement problems.

Performance Expectations:

- MC.3.1.1 Develops the Pythagorean theorem by investigating right triangles, their measures, and related areas.
- MC.3.1.2 Applies the Pythagorean theorem to solve measurement problems.

Objective MC.3.2:

Student applies the concepts of surface area and volume to measure and find formulas for measures of three-dimensional figures.

Performance Expectations:

- MC.3.2.1 Investigates and describes the relationships between the measurements of three-dimensional figures and the measures of related two-dimensional figures (e.g., $A = l \cdot w$, $V = (l \cdot w) \cdot h$).
- MC.3.2.2 Investigates, conjectures, and applies formulas for determining the surface areas and volumes of rectangular solids, other prisms, and cylinders.

- MC.3.2.3 Formulates and applies general statements relating scale changes in the linear dimensions of a figure to the resulting changes in perimeter, area, circumference, surface area, and volume of the resulting figure.

Standard MC.4: Linear Equations and Inequalities

Students develop and apply the connections between rate of change and linear relationships. They create and solve linear equations using tabular and graphical displays, verbal representations of problems, and symbolic manipulation. Students interpret inequalities involving one variable.

Objective MC.4.1:

Student interprets rate of change in real-world and in mathematical settings and recognizes the constant rate of change associated with linear relationships.

Performance Expectations:

- MC.4.1.1 Demonstrates that the rate of change in linear settings is constant, and graphically describes the proportional relationship embedded in this rate of change and represented in the “steepness” or inclination of the related line.
- MC.4.1.2 Interprets, describes, and uses rates of change to model mathematical and real-world situations, noting the different patterns resulting from linear (arithmetic) and exponential (geometric) patterns.
- MC.4.1.3 Connects and translates among equivalent representations of linear relationships, including coordinate graphs and their related tables of values, equations, and verbal rules, to solve problems involving linear patterns.

Objective MC.4.2:

Student creates one- and two-step linear equations and solves such equations using tables, coordinate graphs, and symbolic manipulation.

Performance Expectations:

- MC.4.2.1 Represents linear mathematical and real-world situations using a linear equation of the form $ax + b = c$, where a , b , and c are rational numbers expressed as fractions, decimals, or integers.
- MC.4.2.2 Solves linear equations with rational-number coefficients using mental, graphical, and symbolic methods, with and without technology.

- MC.4.2.3 Connects the graphical, tabular, and symbolic representations of the unique solution of a given linear equation.

Objective MC.4.3:

Student represents and interprets inequalities in one variable geometrically and symbolically.

Performance Expectations:

- MC.4.3.1 Represents solutions to inequalities such as $x > a$ and $a \leq x \leq b$ on a number line.
- MC.4.3.2 Writes an inequality to represent an interval or ray, with or without endpoints, shown on the number line.

Standard MC.5: Bivariate Data

Students formulate and answer questions about a small population by collecting, organizing, and analyzing bivariate data from the population. Students summarize and represent bivariate data. Students communicate the results of data analyses and identify flaws in faulty or misleading presentations of bivariate data found in the media.

Objective MC.5.1:

Student formulates questions about a small population that can be answered through collection and analysis of bivariate data, designs related data investigations, and collects data.

Performance Expectations:

- MC.5.1.1 Formulates a simple question involving two attributes, and defines a small population on which data can be collected to answer the question.
- MC.5.1.2 Identifies two attributes on which to collect data, decides how to measure the two attributes to answer the question formulated, determines a data collection process, and collects data.

Objective MC.5.2:

Student organizes and summarizes bivariate data, examining data on the two attributes separately and together, and classifies each attribute as a categorical variable or numerical variable. Student uses summary statistics and a variety of graphical displays to represent the data.

Performance Expectations:

- MC.5.2.1 Classifies each attribute as corresponding to a categorical or numerical variable. Describes the distribution for each attribute separately using appropriate graphs, including stem-and-leaf plots, histograms, and box plots, and summary statistics, including interquartile range.
- MC.5.2.2 Identifies, describes, and constructs appropriate displays for bivariate data: two-way tables for two categorical variables; parallel box plots or back-to-back stem-and-leaf plots for one numerical and one categorical variable; and scatterplots, with an appropriately sketched trend line, for two numerical variables.
- MC.5.2.3 Explains the usefulness of different displays for bivariate data. Describes the relationship between the two variables and the effects of outliers on the observed relationship.

Objective MC.5.3:

Student interprets results and communicates conclusions from a bivariate data analysis to answer the formulated question using appropriate symbols, notation, and terminology. Student identifies flaws in faulty or misleading presentations of bivariate data found in the media.

Performance Expectations:

- MC.5.3.1 Interprets results and communicates conclusions from a bivariate data analysis in the context of the formulated question using appropriate symbols, notation, and terminology.
- MC.5.3.2 Identifies flaws in faulty or misleading presentations of bivariate data found in the media and their potential effects on conclusions drawn.

Standard MC.6: Probabilities in One-Stage Experiments

Students determine the sample space for one-stage experiments and determine the experimental and, where possible, theoretical probabilities for events defined on the sample space. Students describe probabilities of events governed by the addition rule for probabilities.

Objective MC.6.1:

Student determines the sample space for one-stage experiments and determines, where possible, the theoretical probabilities for events defined on the sample space. Student describes and applies the addition rule for probabilities.

Performance Expectations:

- MC.6.1.1 Determines the sample space for a given one-stage experiment, and uses lists, tables, and tree diagrams to represent all possible outcomes.
- MC.6.1.2 Identifies events for a given sample space, represents relationships among events using Venn diagrams, and determines the probabilities of events and their complements.
- MC.6.1.3 Describes and applies the addition rule for probabilities for events that are mutually exclusive and for events that are not.