



AP[®] Calculus BC 2004 Scoring Guidelines Form B

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Question 1

A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t , where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \quad \text{and} \quad \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t . At time $t = 0$, the particle is at the point $(4, 1)$.

- (a) Find the speed of the particle and its acceleration vector at time $t = 0$.
 (b) Find an equation of the line tangent to the path of the particle at time $t = 0$.
 (c) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.
 (d) Find the x -coordinate of the position of the particle at time $t = 3$.

- (a) At time $t = 0$:

$$\text{Speed} = \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$$

$$\text{Acceleration vector} = \langle x''(0), y''(0) \rangle = \langle 0, -3 \rangle$$

$$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration vector} \end{cases}$$

(b) $\frac{dy}{dx} = \frac{y'(0)}{x'(0)} = \frac{7}{3}$

$$\text{Tangent line is } y = \frac{7}{3}(x - 4) + 1$$

$$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \end{cases}$$

(c) Distance = $\int_0^3 \sqrt{(\sqrt{t^4 + 9})^2 + (2e^t + 5e^{-t})^2} dt$
 = 45.226 or 45.227

$$3 : \begin{cases} 2 : \text{distance integral} \\ \langle -1 \rangle \text{ each integrand error} \\ \langle -1 \rangle \text{ error in limits} \\ 1 : \text{answer} \end{cases}$$

(d) $x(3) = 4 + \int_0^3 \sqrt{t^4 + 9} dt$
 = 17.930 or 17.931

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

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Question 2

Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by $T(x) = 7 - 9(x - 2)^2 - 3(x - 2)^3$.

- (a) Find $f(2)$ and $f''(2)$.
- (b) Is there enough information given to determine whether f has a critical point at $x = 2$?
 If not, explain why not. If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
- (c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether f has a critical point at $x = 0$? If not, explain why not. If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
- (d) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)| \leq 6$ for all x in the closed interval $[0, 2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.

(a) $f(2) = T(2) = 7$
 $\frac{f''(2)}{2!} = -9$ so $f''(2) = -18$

2 : $\begin{cases} 1 : f(2) = 7 \\ 1 : f''(2) = -18 \end{cases}$

(b) Yes, since $f'(2) = T'(2) = 0$, f does have a critical point at $x = 2$.
 Since $f''(2) = -18 < 0$, $f(2)$ is a relative maximum value.

2 : $\begin{cases} 1 : \text{states } f'(2) = 0 \\ 1 : \text{declares } f(2) \text{ as a relative maximum because } f''(2) < 0 \end{cases}$

(c) $f(0) \approx T(0) = -5$
 It is not possible to determine if f has a critical point at $x = 0$ because $T(x)$ gives exact information only at $x = 2$.

3 : $\begin{cases} 1 : f(0) \approx T(0) = -5 \\ 1 : \text{declares that it is not possible to determine} \\ 1 : \text{reason} \end{cases}$

(d) Lagrange error bound $= \frac{6}{4!}|0 - 2|^4 = 4$
 $f(0) \leq T(0) + 4 = -1$
 Therefore, $f(0)$ is negative.

2 : $\begin{cases} 1 : \text{value of Lagrange error bound} \\ 1 : \text{explanation} \end{cases}$

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Question 3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

(a) Midpoint Riemann sum is
 $10 \cdot [v(5) + v(15) + v(25) + v(35)]$
 $= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$
 The integral gives the total distance in miles that the plane flies during the 40 minutes.

3 : $\begin{cases} 1 : v(5) + v(15) + v(25) + v(35) \\ 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t .

2 : $\begin{cases} 1 : \text{two instances} \\ 1 : \text{justification} \end{cases}$

(c) $f'(23) = -0.407$ or -0.408 miles per minute²

1 : answer with units

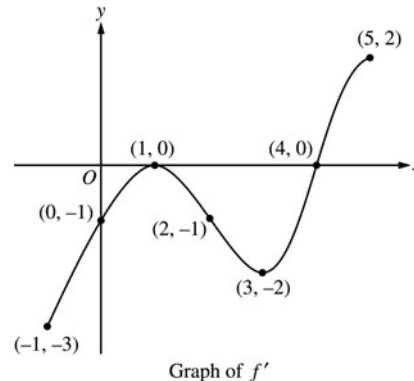
(d) Average velocity $= \frac{1}{40} \int_0^{40} f(t) dt$
 $= 5.916$ miles per minute

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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Question 4

The figure above shows the graph of f' , the derivative of the function f , on the closed interval $-1 \leq x \leq 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.



- (a) Find the x -coordinate of each of the points of inflection of the graph of f . Give a reason for your answer.
- (b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \leq x \leq 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \leq x \leq 5$? Show the analysis that leads to your answers.
- (c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

- (a) $x = 1$ and $x = 3$ because the graph of f' changes from increasing to decreasing at $x = 1$, and changes from decreasing to increasing at $x = 3$.

$$2 : \begin{cases} 1 : x = 1, x = 3 \\ 1 : \text{reason} \end{cases}$$

- (b) The function f decreases from $x = -1$ to $x = 4$, then increases from $x = 4$ to $x = 5$. Therefore, the absolute minimum value for f is at $x = 4$. The absolute maximum value must occur at $x = -1$ or at $x = 5$.

$$4 : \begin{cases} 1 : \text{indicates } f \text{ decreases then increases} \\ 1 : \text{eliminates } x = 5 \text{ for maximum} \\ 1 : \text{absolute minimum at } x = 4 \\ 1 : \text{absolute maximum at } x = -1 \end{cases}$$

$$f(5) - f(-1) = \int_{-1}^5 f'(t) dt < 0$$

Since $f(5) < f(-1)$, the absolute maximum value occurs at $x = -1$.

- (c) $g'(x) = f(x) + xf'(x)$
 $g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$
 $g(2) = 2f(2) = 12$

$$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$$

Tangent line is $y = 4(x - 2) + 12$

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Question 5

Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

- (a) Find the average value of g on the closed interval $[1, 4]$.
- (b) Let S be the solid generated when the region bounded by the graph of $y = g(x)$, the vertical lines $x = 1$ and $x = 4$, and the x -axis is revolved about the x -axis. Find the volume of S .
- (c) For the solid S , given in part (b), find the average value of the areas of the cross sections perpendicular to the x -axis.
- (d) The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be

$\lim_{b \rightarrow \infty} \left[\frac{\int_a^b f(x) dx}{b - a} \right]$. Show that the improper integral $\int_4^{\infty} g(x) dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.

(a) $\frac{1}{3} \int_1^4 \frac{1}{\sqrt{x}} dx = \frac{1}{3} \cdot 2\sqrt{x} \Big|_1^4 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{antidifferentiation} \\ \text{and evaluation} \end{array} \right.$

(b) Volume = $\pi \int_1^4 \frac{1}{x} dx = \pi \ln x \Big|_1^4 = \pi \ln 4$

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{antidifferentiation} \\ \text{and evaluation} \end{array} \right.$

(c) The cross section at x has area $\pi \left(\frac{1}{\sqrt{x}} \right)^2 = \frac{\pi}{x}$

1 : answer

Average value = $\frac{1}{3} \int_1^4 \frac{\pi}{x} dx = \frac{1}{3} \pi \ln 4$

(d) $\int_4^{\infty} g(x) dx = \lim_{b \rightarrow \infty} \int_4^b \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow \infty} (2\sqrt{b} - 4) = \infty$

This limit is not finite, so the integral is divergent.

$$\frac{\int_4^b g(x) dx}{b - 4} = \frac{1}{b - 4} \int_4^b \frac{1}{\sqrt{x}} dx = \frac{2\sqrt{b} - 4}{b - 4}$$

$$\lim_{b \rightarrow \infty} \frac{2\sqrt{b} - 4}{b - 4} = 0$$

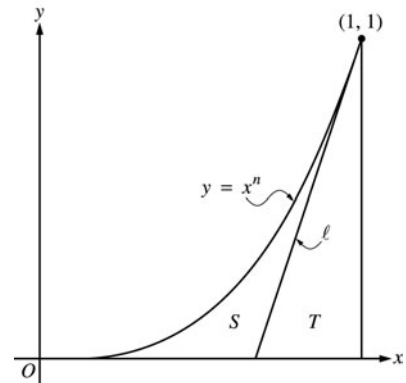
4 : $\left\{ \begin{array}{l} 1 : \int_4^b g(x) dx = 2\sqrt{b} - 4 \\ 1 : \text{indicates integral diverges} \\ 1 : \frac{1}{b - 4} \int_4^b g(x) dx = \frac{2\sqrt{b} - 4}{b - 4} \\ 1 : \text{finite limit as } b \rightarrow \infty \end{array} \right.$

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Question 6

Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

- (a) Find $\int_0^1 x^n dx$ in terms of n .
- (b) Let T be the triangular region bounded by ℓ , the x -axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.
- (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ , and the x -axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S .



(a) $\int_0^1 x^n dx = \frac{x^{n+1}}{n+1} \Big|_0^1 = \frac{1}{n+1}$

2 : $\left\{ \begin{array}{l} 1 : \text{antiderivative of } x^n \\ 1 : \text{answer} \end{array} \right.$

- (b) Let b be the length of the base of triangle T .

$\frac{1}{b}$ is the slope of line ℓ , which is n

3 : $\left\{ \begin{array}{l} 1 : \text{slope of line } \ell \text{ is } n \\ 1 : \text{base of } T \text{ is } \frac{1}{n} \\ 1 : \text{shows area is } \frac{1}{2n} \end{array} \right.$

$$\text{Area}(T) = \frac{1}{2}b(1) = \frac{1}{2n}$$

(c) $\text{Area}(S) = \int_0^1 x^n dx - \text{Area}(T)$
 $= \frac{1}{n+1} - \frac{1}{2n}$

4 : $\left\{ \begin{array}{l} 1 : \text{area of } S \text{ in terms of } n \\ 1 : \text{derivative} \\ 1 : \text{sets derivative equal to } 0 \\ 1 : \text{solves for } n \end{array} \right.$

$$\frac{d}{dn} \text{Area}(S) = -\frac{1}{(n+1)^2} + \frac{1}{2n^2} = 0$$

$$2n^2 = (n+1)^2$$

$$\sqrt{2}n = (n+1)$$

$$n = \frac{1}{\sqrt{2}-1} = 1 + \sqrt{2}$$