Section 4: Mathematics

Mathematics Question 21

Choice (B) is correct. The equation $6u = 4t$ gives 6u in terms of t. Since the question asks you to find 3u, you can divide both sides of the equation by 2 to get $\frac{6u}{2} = \frac{4t}{2}$, which simplifies to $3u = 2t$.

Mathematics Question 22

Choice (E) is correct. In this question you are given a definition for $f(x)$ and are asked to find the value of $f(7)$. Substituting 7 for x gives $f(7) = \frac{7+1}{7-1} = \frac{8}{6}$.

Mathematics Question 23

Choice (C) is correct. A passenger making a continuous one-way trip on the railroad could travel a total distance, in miles, of any of the distances shown on the figure; namely, 2, 3, or 4, or any combination of distances of adjacent sections of the railroad route, namely, $2 + 3 = 5$, $3 + 4 = 7$, and $2 + 3 + 4 = 9$. So the possible distances, in miles, are 2, 3, 4, 5, 7, and 9. Among the answer choices, the only one that is not a possible distance is 6. Note that a total distance of 6 cannot be obtained by adding the 2 miles between Ashton and Bell and the 4 miles between Carson and Duncan, because these sections of the route are not adjacent to each other.

Mathematics Question 24

Choice (A) is correct. The positive 2-digit integers that have both digits the same are 11, 22, 33, 44, 55, 66, 77, 88, and 99. There are 9 such integers.

Mathematics Question 25

Choice (A) is correct. Since the average of x, x, and y is x, you can write the equation $(x + x + y) \div 3 = x$, which is equivalent to $2x + y = 3x$, which gives $y = x$. Answer choice (A), $x = y$, is equivalent to this result. Since $x = y$, answer choices (B) and (C) cannot be true. Answer choices (D) and (E) are true only if x and y both equal 0. The question asks which statement <u>must</u> be true. Since these last two answer choices do not have to be true, (D) and (E) do not qualify as statements that must be true.

Mathematics Question 26

Choice (B) is correct. If you let x stand for the number of children who received exactly one balloon, then $150 - x$ is the number of children who received two balloons. The number of balloons given out can be expressed as $x + 2(150 - x)$, which can be simplified to $300 - x$. The question tells you that the total number of balloons distributed was 215, so you can write the equation $300 - x = 215$. From this it follows that $x = 300 - 215 = 85$.

It is also possible to answer this question without introducing a variable. Since each child received at least one balloon, and since there were 150 children, the clown used up 150 balloons giving each child just one. This would leave $215 - 150 = 65$ balloons to distribute. Each child who received one of these 65 balloons would have received two balloons, so there were 65 children who received two balloons. Therefore, the number of children who received just one balloon was $150 - 65 = 85$.

Mathematics Question 27

Choice (D) is correct. The expression $k^{\frac{1}{3}} \cdot k^{\frac{1}{3}} \cdot k^{\frac{1}{3}}$ can be simplified to $\left(k^{\frac{1}{3}}\right)^3$, which is equivalent to k. Substituting k for the left side of the equation given in the question yields $k = 1$. The question asks which expression is equivalent to x^k, so you can substitute 1 for k and get x^1, which is equivalent to x.

Mathematics Question 28

Choice (B) is correct. You may find it helpful to draw a picture of line ℓ in the xy-plane and then draw line m, the reflection of ℓ across the x-axis. You can rule out the cases where line ℓ is parallel to the x-axis because in those cases lines ℓ and m will both be parallel to the x-axis and will be parallel to each other. (Note that the question indicates that the two lines intersect.) You can also rule out the cases where line ℓ is parallel to the y-axis, because then the reflection of line ℓ across the x-axis would be itself, and so the two lines ℓ and m would be the same line and their intersection would consist of all the points on line ℓ. (Note that the question says that the intersection is the point (r, s).)

If you draw line ℓ so that it crosses the x-axis at $(-2, 0)$, you will see that the point of intersection of lines ℓ and m is also the point $(-2, 0)$. Similarly, if you draw line ℓ so that it crosses the x-axis at $(5, 0)$, you will see that the point of intersection is also $(5, 0)$. In fact, the point (r, s) must be on the x-axis no matter how you draw line ℓ. This means that r can have any value, but s has only one possible value, 0. Answer choices (A), (C), and (D) will be true if line ℓ goes through the origin, but they do not qualify as statements that must be true. Answer choice (E) cannot be true, since $s = 0$, and therefore $rs = 0$.

Mathematics Question 29

The correct answer is 33.5 or 67/2. Since the sum of the degree measures of the angles of a triangle is 180, you can write the equation $43 + 70 + 2y = 180,$ which is equivalent to $113 + 2y = 180,$ or $2y = 67.$ From this you can see that y must be 67/2, or 33.5.

Mathematics Question 30

The correct answer is 2. If x stands for the number of times during the hour that Shannon pushed the button and y stands for the number of times she turned the switch, you know that $7x + 3y = 23.$ The values of x and y must be positive integers, so to find the values of x and $y,$ you need to find a positive multiple of 3 and a positive multiple of 7 that add up to 23.

If $x = 1,$ then $3y = 23 - 7 = 16.$ Since 16 is not a multiple of 3, this is not a solution. If $x = 2,$ then $3y = 23 - 14 = 9.$ Since 9 is a multiple of 3, this solution works. If $x = 3,$ then $3y = 23 - 21 = 2.$ Since 2 is not a multiple of 3, this is not a solution. For larger values of $x,$ the value of y would have to be negative, and that is inconsistent with the situation described in the question. The only possible value of x for this situation is 2. The equation would be $7(2) + 3(3) = 23.$

Mathematics Question 31

The correct answer is 0. Since the ratio of x to y is given as 3 to 2, or $\frac{x}{y} = \frac{3}{2},$ it is possible to express x as $\frac{3}{2}y.$ Substituting $\frac{3}{2}y$ for x in the expression $4x^2 - 9y^2$ gives $4 \cdot \frac{9}{4}y^2 - 9y^2,$ which is equivalent to $9y^2 - 9y^2,$ or 0.

Mathematics Question 32

The correct answer is 10. When a number of nails weighing 1500 grams were removed from the jar, there were 850 nails left. This means that the number of nails removed was $1000 - 850 = 150.$ From this you know that 150 nails weigh 1500 grams. This means that each nail weighs $1500 \div 150 = 10$ grams.

Mathematics Question 33

The correct answer is 82. Since $\frac{x}{y} = 2,$ it follows that $y = \frac{x}{2}.$ You also know that $x + y = 123.$ Substituting $\frac{x}{2}$ for y in this equation gives $x + \frac{x}{2} = 123.$ Multiplying both sides of this equation by 2 gives $2x + x = 246,$ which means that $3x = 246,$ and so $x = 82.$

Mathematics Question 34

The correct answer is 1050. The percents given for three of the sectors of the circle graph are 50%, 35%, and 8%. The sum of these three percents is 93%. Since the entire graph summarizes 100% of the results of the survey, the sector labeled "Other" must be $100\% - 93\%$, or 7%. Since 5250 students had summer jobs in recreation, and since that group is 35% of the total, you can conclude that 5250 is 35% of the total number of students. The group that answered "Other" makes up 7% of the total. Since 7% is $\frac{1}{5}$ of 35%, the number of students in that category is $\frac{1}{5}$ of 5250, which is $5250 \div 5$, or 1050.

Another approach to this question involves finding the total number of students who responded to the survey and then taking 7% of that total. If 35% of the total is 5250, then $0.35t = 5250$, where t is the total number of students. Solving for t gives $5250 \div 0.35$, which equals 15,000. The number of students in the "Other" category is 7% of 15,000, which is 1050.

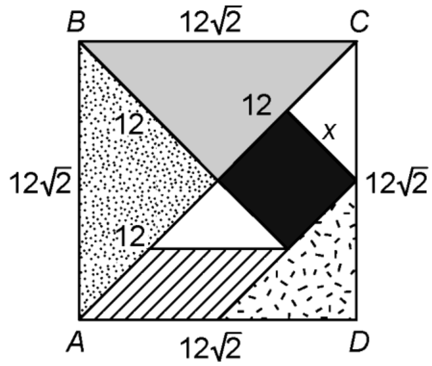
Mathematics Question 35

The correct answer is any number greater than 0 and less than 1/2. Since the square of t is less than one half of t, it follows that t must be positive. (If t were negative, then half of t would also be negative but its square would be positive, and this would contradict the statement that the square of t is less than one half of t. If t were equal to 0, then the square of t and one half of t would both equal 0, and this also would contradict the statement that the square of t is less than one half of t.)

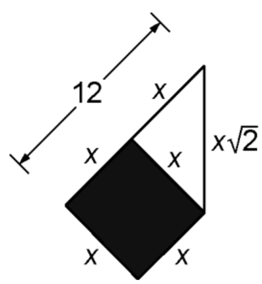
The statement given in the question can be expressed by the inequality $t^2 < \frac{t}{2}$. Since t is positive, you can divide both sides of the inequality by t, which gives $t < \frac{1}{2}$. Putting the two conditions together, namely, that t is greater than 0 and t is less than $\frac{1}{2}$, gives the statement $0 < t < 1/2$. Once you get this answer, you should grid in any number greater than 0 but less than $1/2$. For example, you could grid in the number $1/3$. To check this value, notice that $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ and that one half of $\frac{1}{3}$ is $\frac{1}{6}$. Since $\frac{1}{9} < \frac{1}{6}$, the number 1/3 is a possible correct answer.

Mathematics Question 36

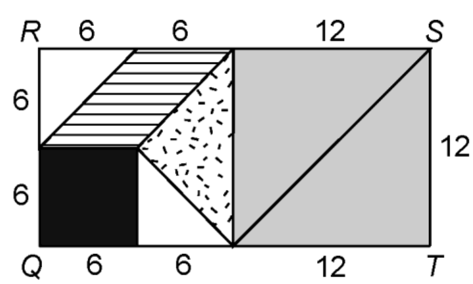
The correct answer is 72. Square $ABCD$ has sides of length $12\sqrt{2}$, so each diagonal of the square has length $12\sqrt{2}(\sqrt{2})$, which is 24. Triangle ABC has two sides of length $12\sqrt{2}$ and a hypotenuse of length 24. This triangle is divided into two isosceles right triangles, each with sides of length 12 and hypotenuse of length $12\sqrt{2}$. Let x stand for the length of each side of the dark square. At this point, the information about the figure on the left can be summarized as follows:



Since the dark square has all its sides of equal length, you can label each of these sides x. The white isosceles right triangles on either side of the square each share a side with the small dark square, so each of these triangles must have legs of length x and hypotenuses of length $x\sqrt{2}$. The figure below shows the dark square and one of the white triangles, with more lengths labeled.



From this you can see that $x + x = 12$, and therefore $x = 6$. This allows you to label all the line segments in the square. The two largest right triangles have lengths 12, 12, and $12\sqrt{2}$; the two small white triangles have lengths 6, 6, and $6\sqrt{2}$; the medium-sized speckled right triangle has lengths $6\sqrt{2}$, $6\sqrt{2}$, and 12; and the striped parallelogram has sides of lengths 6 and $6\sqrt{2}$. When you transfer the lengths to the corresponding segments in the figure on the right (rectangle *QRST*) the result will be this:



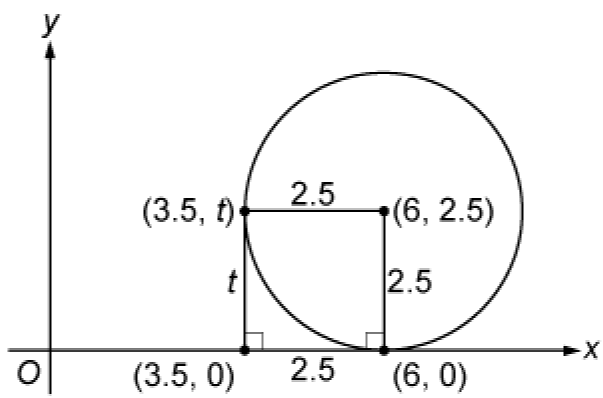
From this you can see that $RS = 24$ and $ST = 12$. Thus, perimeter of $QRST$ is $24 + 12 + 24 + 12 = 72$.

Mathematics Question 37

The correct answer is 144. Since $x^2 = 25,$ there are two possible values of x, namely, 5 and $-5.$ Similarly, the possible values of y are 4 and $-4,$ and the possible values of z are 3 and $-3.$ To get the greatest possible value of $(x + y - z)^2,$ you should make $(x + y - z)$ as great as possible (if it is positive) or as small as possible (if it is negative). To make $(x + y - z)$ as great as possible, choose 5 for x, 4 for y, and -3 for z. Then $(x + y - z)^2$ will equal $(5 + 4 + 3)^2,$ which is 12^2, or 144. The other possibility is to choose -5 for x, -4 for y, and 3 for z, which gives a value of $[(-5) + (-4) - 3]^2,$ which is equal to $(-12)^2,$ or 144. In either case, the maximum value is 144.

Mathematics Question 38

The correct answer is 2.5 or 5/2. It may be helpful to draw a picture of the circle described in the question. Since the circle has exactly one point in common with the x-axis, you should draw the circle so that it is tangent to the x-axis.



The point of intersection of the circle and the x-axis must be directly below the center. Since the center of the circle is $(6, 2.5),$ the point of intersection must be $(6, 0),$ and therefore the radius of the circle is 2.5. (Note that if you can see that this is true, it is not necessary on this test to prove it. It is, however, possible to prove it using the theorem that when a line is tangent to a circle, a radius drawn to the point of tangency must be perpendicular to the tangent line.) Another point on the circle is $(3.5, t).$ This point is directly above $(3.5, 0).$ Since the difference between 6 and 3.5 is 2.5, which is the radius of the circle, it should be clear from your drawing that the points $(6, 2.5),$ $(6, 0),$ $(3.5, 0),$ and $(3.5, t)$ are the vertices of a square. (Again, you do not have to prove this when you are taking the test, but it can be proved.) From this it follows that $t = 2.5.$

Another approach is to use the fact that the radius of the circle is 2.5 and the center of the circle is $(6, 2.5)$. Since $(3.5, t)$ is a point on the circle, its distance from $(6, 2.5)$ is 2.5. Using the distance formula, you can translate this information into the equation $(2.5)^2 = (3.5 - 6)^2 + (t - 2.5)^2$, which is equivalent to $(2.5)^2 = (-2.5)^2 + (t - 2.5)^2$. Since $(-2.5)^2$ is equal to $(2.5)^2$, this equation simplifies to $0 = (t - 2.5)^2$, which means that $(t - 2.5) = 0$, and therefore $t = 2.5$.