Section 2: Mathematics

Mathematics Question 1

Choice (C) is correct. Since it is given that $b = 6,$ you can substitute 6 for b in the first equation. This gives $a + 6 = 16.$ From this it follows that $a = 10.$ Since $a = 10$ and $b = 6,$ the value of $a - b$ is $10 - 6,$ which is 4.

Mathematics Question 2

Choice (B) is correct. When you round a number to the nearest tenth, there will be no digits to the right of the tenths place. Choices (D) and (E), therefore, cannot be correct. In the number given, the digit in the hundredths place, 6, is greater than 5, so rounding will involve increasing the tenths digit by 1. Therefore, the correct rounded form is 2.1.

Mathematics Question 3

Choice (E) is correct. The first four terms of the sequence are given, as well as the rule for producing all terms after the first. Since the fourth term is 2, it follows that the fifth term is half of 2, which is 1; the sixth term is half of 1, which is $\frac{1}{2};$ and the seventh term is half of $\frac{1}{2},$ which is $\frac{1}{4}.$

Mathematics Question 4

Choice (B) is correct. The probability of selecting a green shirt from the drawer is the number of green shirts divided by the total number of shirts in the drawer. The total number of shirts is $10 + 6 + 2,$ which is 18. The number of green shirts is 2. Therefore, the probability of selecting a green shirt is $\frac{2}{18},$ which is equal to $\frac{1}{9}.$

Mathematics Question 5

Choice (A) is correct. In trapezoid ABCD the two parallel sides are $\overline{BC}$ and $\overline{AD}.$ These two parallel sides are cut by side $\overline{AB},$ and this means that the sum of 75 and x is 180. From the equation $75 + x = 180,$ you can conclude that $x = 105.$

Alternatively, you could use the fact that the sum of the degree measures of the four angles in any quadrilateral must be 360. This gives the equation $75 + 75 + x + x = 360,$ which is equivalent to the equation $75 + x = 180.$

Mathematics Question 6

Choice (C) is correct. On a number line, the number halfway between two numbers is the average (i.e., the arithmetic mean) of the two numbers. So, to find the number halfway between $\frac{1}{2}$ and $\frac{3}{4}$, add $\frac{1}{2}$ and $\frac{3}{4}$ and divide the sum by 2. This gives $\frac{1}{2}\left(\frac{2}{4}+\frac{3}{4}\right)$, which equals $\frac{1}{2}\left(\frac{5}{4}\right)$, which is $\frac{5}{8}$.

Another way to answer this question is to find the distance between $\frac{1}{2}$ and $\frac{3}{4}$, divide that distance by 2, and then either add that amount to $\frac{1}{2}$ or subtract that amount from $\frac{3}{4}$. The distance between $\frac{3}{4}$ and $\frac{1}{2}$ is $\frac{3}{4}-\frac{1}{2}$, which is $\frac{1}{4}$. Half of $\frac{1}{4}$ is $\frac{1}{8}$. When you add $\frac{1}{8}$ to $\frac{1}{2}$, you get $\frac{5}{8}$; alternatively, when you subtract $\frac{1}{8}$ from $\frac{3}{4}$, you get $\frac{5}{8}$.

Mathematics Question 7

Choice (E) is correct. Count the boxes in the graph that represent the students who received a grade of 60 or more: there are 5 boxes in the column for 60 - 69; 6 boxes in the column for 70 - 79; 4 boxes in the column for 80 - 89; and 1 box in the column for 90 - 99; that is, there are $5+6+4+1=16$ such boxes. Then count the number of boxes that represent the students who received a grade of 59 or less: 3 boxes in the column for 50 - 59; 2 boxes in the column for 40 - 49; 1 box in the column for 30 - 39; and 1 box in the column for 20 - 29; that is, there are $3+2+1+1=7$ such boxes. The difference between 16 boxes and 7 boxes is 9 boxes. Each box represents 10 students, so the difference between the number of students in the two groups is 10 times 9, which is 90.

Mathematics Question 8

Choice (A) is correct. In this question you are given three expressions (labeled I, II, and III) and are asked which of them must be negative. The expressions involve variables u and t, and you are told that u and t are negative integers. For this type of question, you should consider each of the three expressions separately and determine whether the expression must be negative.

Expression I, $u+t$, must be negative, because whenever you add two negative integers, the sum is a negative integer. Expression II, $u-t$, could be either negative or positive or zero. If u is greater than t, then $u-t$ is positive. For example, if $u=-3$ and $t=-5$, then $u-t=-3-(-5)=-3+5=2$. If u is less than t, then $u-t$ is negative. For example, if $t=-4$, and $u=-10$, then $u-t=-10-(-4)=-10+4=-6$. If u is equal to t, then their difference is zero. Since expression II can be negative but does not have to be negative, expression II does not qualify as one that must be negative. Expression III is always positive because the product of two negative numbers is always positive, so expression III also does not qualify as one that must be negative. Of the three expressions, only expression I must be negative.

Mathematics Question 9

Choice (B) is correct. In the triangle, the measure of one angle is 90 degrees (the angle marked with a right-angle symbol) and the measure of another angle is 45 degrees. Since the sum of the measures of the angles of a triangle is 180 degrees, the measure of the angle at W must be 45 degrees. This triangle is a 45-45-90 triangle, one in which the length of the hypotenuse is $\sqrt{2}$ times the length of either one of the other sides. Since the length of side $\overline{VW}$ is 6, the length of the hypotenuse is $6\sqrt{2}$. (The information about 45-45-90 triangles is given in the reference information at the beginning of the section.)

Alternatively, you could solve this question by applying the Pythagorean Theorem. Use the fact that the length of side $\overline{TV}$ must also be 6 because angles T and W are congruent. From this you can compute the length of the hypotenuse as $\sqrt{6^2 + 6^2}$, which is $\sqrt{36 + 36}$, or $\sqrt{72}$. Since $72 = 2 \cdot 36$, it follows that $\sqrt{72}$ is equal to $\sqrt{2} \cdot \sqrt{36}$, which is equal to $6\sqrt{2}$.

Mathematics Question 10

Choice (A) is correct. The words "Let $a,b \ \Delta n$ be defined" tell you that the triangle symbol in this question is being defined for the question. This is not an ordinary operation in arithmetic, and you are not expected to know this definition. Since the question asks for the value of $(10,10)n\ 2$, you should substitute 10 for a, 10 for b, and 2 for n in the definition. The result is $\frac{10^2(10) + 10\left(10^2\right)}{10^2}$. Since 10^2 is a factor of each term in the numerator and in the denominator of this fraction, you can simplify the fraction by dividing each term by 10^2. This makes the expression $\frac{10 + 10}{1}$, which equals $10 + 10$, or 20.

Mathematics Question 11

Choice (C) is correct. Since the students arranged themselves in 8 rows with the same number of students in each row, you know that the number of students is a multiple of 8. If p represents the number of students in each row, then the total number of students in the class is 8p. When the students arranged themselves into 9 rows, there were 3 fewer students per row than when they arranged themselves into 8 rows, so another way to express the total number of students in the class is $9(p - 3)$. These two expressions for the same quantity (total number of students in the class) give rise to the equation $8p = 9(p - 3)$, which is equivalent to $8p = 9p - 9 \cdot 3$. Solving this equation results in

$p = 27.$ Therefore, the total number of students in the class is $8 \cdot 27,$ which is 216.

Notice that when the students arrange themselves into 9 rows, there are 3 fewer students, or 24 students, in each row. The product $9 \cdot 24$ also equals 216.

Mathematics Question 12

Choice (B) is correct. One way to approach this question is to let n stand for the middle number of the three consecutive odd integers. Then the least integer is $n - 2$ and the greatest is $n + 2.$ The sum of all three integers is then $(n - 2) + n + (n + 2)$, which is equivalent to 3n. The sum of the least and the greatest of the integers is $(n - 2) + (n + 2),$ which is equivalent to 2n.

You are given that the sum of the least and the greatest of the integers is 146, so $2n = 146;$ therefore, $n = 73.$ The question asks for the sum of all three integers, which is 3n, so this sum equals $3 \times 73,$ which is 219. (Once you have found that $n = 73,$ you can also actually find the other two integers. Since $n = 73,$ it follows that $n - 2 = 71$ and $n + 2 = 75.$ The sum $71 + 73 + 75$ equals 219.)

Mathematics Question 13

Choice (E) is correct. The figure for this question includes two triangles, one inside the other. In the smaller triangle, one angle is labeled 50° and another is labeled $x^\circ.$ When parallel lines are cut by a transversal, alternate interior angles must be congruent. So the third angle in this triangle must have a measure of $60^\circ.$ Since the sum of the measures of the angles in a triangle is $180^\circ,$ you can conclude that $50^\circ + 60^\circ + x^\circ = 180^\circ,$ which is equivalent to $110^\circ + x^\circ = 180^\circ.$ From this you can see that x must be 70.

Mathematics Question 14

Choice (C) is correct. This question asks for an expression representing the profit made by the club. Recall that profit is equal to total income minus total cost. Since each candy bar was sold for $0.75 and n candy bars were sold, the total income, in dollars, was 0.75n. To find the total cost, multiply the number of boxes bought by the cost per box. Since the club bought 30 boxes at a cost of $30.00 each, the total cost, in dollars, was $30 \times 30,$ or 900. The profit is the total income, 0.75n dollars, minus the total cost, 900 dollars, so the expression for the profit, in dollars, is $0.75n - 900.$ It turns out that the information about the number of candy bars per box is not needed to solve the problem.

Mathematics Question 15

Choice (D) is correct. The two lines described in the question are perpendicular, so their slopes are negative reciprocals of each other. Since the slope of line ℓ is 3, the slope of the other line must be $-\frac{1}{3}$. When you look at the answer choices, you may notice that each one is written in the form $y = mx + b$, and you may recall that when the equation of a line is written in this form, the value of m is the slope of the line and the value of b is the y-intercept. Only two of the answer choices, (C) and (D), are equations of lines with a slope of $-\frac{1}{3}$, so the correct answer must be one of these two.

Another piece of information given in the question is that the two lines intersect at the y-intercept of line ℓ. This means that the y-intercept of the second line must be the same as the y-intercept of line ℓ, which is given as 2. Therefore, the equation of the second line is $y = -\frac{1}{3}x + 2$.

It is also possible to approach this question by making a sketch of line ℓ in the xy-plane. It will go through the points $(0, 2)$, $(1, 5)$, $(2, 8)$, and so on. Then sketch a line that goes through $(0, 2)$ and is perpendicular to line ℓ. From your sketch you may be able to eliminate all the answer choices except (D).

Mathematics Question 16

Choice (D) is correct. To find $(r + w)$ in terms of t, you can express r in terms of t and w in terms of t. You are given that $r = \frac{t}{2}$, so you already have r in terms of t. You are also given the equation $t = \frac{w}{2}$, which allows you to solve for w and get $w = 2t$. Substituting $\frac{t}{2}$ for r and $2t$ for w in the expression $(r + w)$ gives $\left(\frac{t}{2} + 2t\right)$, which is equivalent to $\frac{t + 4t}{2}$, or $\frac{5}{2}t$.

Mathematics Question 17

Choice (B) is correct. You are asked to find the total area of the shaded regions. Each of the shaded regions is part of a circle. From the fact that $MNOP$ is a square, you can see that angle ONM and angle OPM are right angles. Therefore, each circle has a fourth of the circle unshaded. (The right angle determines one-fourth of the circle because $90°$ is one-fourth of $360°$.) So, each shaded region is $\frac{3}{4}$ of a circle. The area of each of the whole circles is $\pi\left(4^2\right)$, or 16π. Three-fourths of 16π is 12π. The area of each shaded region is 12π, so the sum of the two areas is $2 \cdot 12\pi$, or 24π.

Mathematics Question 18

Choice (D) is correct. The statement "If $x = 4$, then $y > 3$" is of the form "If p, then q," where p represents the statement "$x = 4$" and q represents the statement "$y > 3$." Whenever a statement of the form "If p then q" is true, then its contrapositive, "If not q, then not p" is also true.

The contrapositive of "If $x = 4$, then $y > 3$" is "If y is not greater than 3, then $x \neq 4$." Based on the statement given in the problem, this contrapositive statement must be true. Notice the statement in choice (D), "If $y < 3$, then $x \neq 4$." If $y < 3$, then y is not greater than 3, and therefore $x \neq 4$. So the statement in choice (D) must be true.

Mathematics Question 19

Choice (A) is correct. It may be helpful to draw a sketch of the cube with point P as the center of one face and point Q as the center of the opposite face. For example, if P is the center of the face closest to the viewer, then Q will be the center of the face farthest from the viewer. To go from P to Q, you could go straight up to the top face of the cube, straight across the top face, and then down from the far edge of the top face to point Q. If the length of the edge of the cube is x, then the length of this path from P to Q is $\frac{x}{2} + x + \frac{x}{2}$, which is equal to 2x. There are other ways to travel along a shortest possible path from P to Q, but they all will have length 2x. Since the length of this path is given as $2\sqrt{2}$, you can write the equation $2x = 2\sqrt{2}$. Solving this gives $x = \sqrt{2}$. The volume of the cube is x^3, which is $\left(\sqrt{2}\right)^3$, or $2\sqrt{2}$.

Mathematics Question 20

Choice (D) is correct. The equation given with this question is true for all values of x; it is also given that a and b are constants. The quantity on the right side of the equation, $(x + b)^2$, can be rewritten as $x^2 + 2bx + b^2$. Since the term x^2 appears on both sides of the equation, the equation can be simplified to $16x + a = 2bx + b^2$. For this equation to be true for all values of x, it must be the case that $16x = 2bx$, which means that $16 = 2b$ and that $b = 8$. Also, it must be the case that $a = b^2$. Since $b = 8$, it follows that $b^2 = 64$; therefore, $a = 64$.

Another way to approach this question is to reason that if the equation is true for all values of x, then it must be true for particular values of x, such as 0 and 1. If $x = 0$, the equation becomes $0^2 + 16 \times 0 + a = (0 + b)^2$, which reduces to $a = b^2$. If $x = 1$, the equation becomes $1^2 + 16 \times 1 + a = (1 + b)^2$, which is $17 + a = 1 + 2b + b^2$. Since $a = b^2$, the latter equation can be further simplified to $17 = 1 + 2b$. The solution to this equation is $b = 8$, and from this it follows that $a = 8^2 = 64$.